THE EQUIVALENT COEFFICIENT OF THERMAL ACTIVITY OF FLOORS

A. G. Gindoyan

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The heat transfer process between a human foot and a floor is examined. It is shown that the parameter determining the insulating quality of the floor is the equivalent coefficient of thermal activity. A method is given for determining this coefficient.

Given an infinite slab of thickness l_1. At the initial moment of time a linear temperature distribution obtains over the cross section of the slab. In the surface plane of the slab (x = 0) the temperature is t_c, and in a plane at depth l_1 ($x = l_1$) t_m. The slab is brought into contact with a semi-infinite body whose initial temperature is constant throughout its volume and equal to t_m. It is required to determine the intensity of the heat flux through the slab surface when a constant temperature t_c is maintained at this surface from the time of contact.

$$\frac{\partial}{\partial \tau} t_{1(x,\tau)} = a_1 \frac{\partial^2}{\partial x^2} t_{1(x,\tau)}, \quad 0 \leqslant x \leqslant l_1 . \tag{1}$$

$$\frac{\partial}{\partial \tau} t_{2(x,\tau)} = a_2 \frac{\partial^2}{\partial x^2} t_{2(x,\tau)}, \quad l_1 \leqslant x < \infty . \tag{2}$$

The initial conditions are:

$$t_{1(x,0)} = t_c - \frac{x}{l_1}(t_c - t_m),$$
$$t_{2(x,0)} = t_m. \tag{3}$$

The boundary conditions are:

$$t_{1(l_1,\tau)} = t_{2(l_1,\tau)},$$
$$\frac{\partial}{\partial x} t_{1(l_1,\tau)} = \frac{\lambda_2}{\lambda_1}\frac{\partial}{\partial x} t_{2(l_1,\tau)}, \tag{4}$$
$$t_{1(0,\tau)} = t_c .$$

The solution of differential equations (1) and (2) with initial and boundary conditions (3) and (4) has the form:

$$t_{1(x,\tau)} = t_c - \frac{x}{l}(t_c - t_m) + \frac{\sqrt{a_1}\,(t_c - t_m)}{l_1(1+\delta)} \sum_{n=1}^{\infty} (-h)^{n-1} \left\{ 2\sqrt{\frac{\tau}{\pi}} \times \right.$$
$$\times \exp\left[-\left(\frac{(2n-1)l_1 - x}{2\sqrt{a_1\tau}}\right)^2\right] - \frac{(2n-1)l_1 - x}{\sqrt{a_1}} \times$$
$$\times \left[\frac{(2n-1)l_1 - x}{2\sqrt{a_1\tau}}\right] - 2\sqrt{\frac{\tau}{\pi}} \exp\left[-\left(\frac{(2n-1)l_1 + x}{2\sqrt{a_1\tau}}\right)^2\right] +$$
$$\left. + \frac{(2n+1)l_1 + x}{\sqrt{a_1}} \operatorname{erfc}\left[\frac{(2n-1)l_1 + x}{2\sqrt{a_1\tau}}\right] \right\}; \tag{5}$$

$$t_{2(x,\tau)} = t_m + \frac{\sqrt{a_1}\,(t_c - t_m)}{l_1(1+\delta)} \sum_{n=1}^{\infty} (-h)^{n-1} \left\{ 2\sqrt{\frac{\tau}{\pi}} \times \right.$$
$$\times \exp\left[-\left(\frac{2(n-1)l_1\sqrt{a_2} - (l_1 - x)\sqrt{a_1}}{2\sqrt{a_1 a_2 \tau}}\right)^2\right] - \frac{2(n-1)l_1\sqrt{a_2} - (l_1 - x)\sqrt{a_1}}{\sqrt{a_1 a_2}} \times$$
$$\times \operatorname{erfc}\left[\frac{2(n-1)l_1\sqrt{a_2} - (l_1 - x)\sqrt{a_1}}{2\sqrt{a_1 a_2 \tau}}\right] - 2\sqrt{\frac{\tau}{\pi}} \times$$

$$\times \exp\left[-\left(\frac{2nl\sqrt{a_2}-(l_1-x)\sqrt{a_1}}{2\sqrt{a_1 a_2 \tau}}\right)^2\right]+\frac{2nl\sqrt{a_2}-(l_1-x)\sqrt{a_1}}{\sqrt{a_1 a_2}}\times$$

$$\times \operatorname{erfc}\left[\frac{2nl_1\sqrt{a_2}-(l_1-x)\sqrt{a_1}}{2\sqrt{a_1 a_2 \tau}}\right]\Bigg\}, \tag{6}$$

where $h=\frac{1-\delta}{1+\delta}$; $\delta=\frac{b_2}{b_1}=\frac{\sqrt{\lambda_2 c_2 \gamma_2}}{\sqrt{\lambda_1 c_1 \gamma_1}}$.

We are interested in the intensity of the heat flux through the slab surface (x = 0):

$$q_{(0,\,\tau)}=-\lambda_1\frac{\partial}{\partial x}t_{1\,(0,\,\tau)}\ . \tag{7}$$

Differentiating $t_{1(x,\tau)}$ with respect to x and substituting in (7), we obtain

$$q_{(0,\,\tau)}=\frac{2\lambda_1(t_c-t_m)}{l_1(1+\delta)}\sum_{n=1}^{\infty}(-h)^{n-1}\operatorname{erf}\left[\frac{(2n-1)l_1}{2\sqrt{a_1\tau}}\right]. \tag{8}$$

Writing $R_1=\frac{l_1}{\lambda_1}$, $k=\frac{l_1}{2\sqrt{a_1\tau}}=\frac{1}{2\sqrt{\mathrm{Fo}_1}}$ and $\varphi_{1\,(\delta,\,k)}=\frac{2}{1+\delta}\times\sum_{n=1}^{\infty}(-h)^{n-1}\operatorname{erf}[(2n-1)k]$, we obtain

$$q_{(0,\,\tau)}=\frac{t_c-t_m}{R_1}\varphi_{1\,(\delta,\,k)}\,. \tag{9}$$

Values of $\varphi_{1(\delta,k)}$ are given in Fig. 1 as a function of $\delta=b_2/b_1$ and $k=1/2\sqrt{\mathrm{Fo}_1}$.

The total amount of heat $Q_{(0,\tau)}$ passing through the slab surface (x = 0) in the time interval from τ = 0 to $\tau=\tau_1$ is determined by the integral:

$$Q_{(0,\,\tau)}=\int_0^{\tau_1}q_{(0,\,\tau)}\,d\tau. \tag{10}$$

Substituting the value of $q_{(0,\tau)}$ from (8) in (10) and integrating, we obtain

$$Q_{(0,\,\tau)}=\frac{t_c-t_m}{R}\,\frac{2\tau}{\sqrt{\pi}}\,\frac{4}{1+\delta}\sum_{n=1}^{\infty}(-h)^{n-1}\times$$

$$\times\left\{(2n-1)k\sum_{\eta=0}^{\infty}(-1)^{\eta+1}\frac{[(2n-1)k]^{2\eta}}{\eta!\,(2\eta-1)(2\eta+1)}\right\}. \tag{11}$$

Analysis of the expressions obtained shows that for contact with shod feet, the heat flux and the total amount of heat absorbed depend on the coefficient of thermal activity of the floor material. Hence for a given kind of footwear the heat losses of a foot are determined by this coefficient.

The design value of the coefficient of thermal activity of floors in the main rooms of dwellings and public buildings must be established on the basis of the maximum permissible heat loss for the feet, as determined by special hygienic research.

Equations (8) and (11) were derived for the case of an infinite floor thickness. It is evident that the temperature changes will be transmitted gradually to the interior of the floor. Therefore, up to some known moment of time they will be practically the same as the changes in the wall. In practice, for an underlying layer 2-3 cm thick and contact times τ = 0. 20 hr, we may assume in the calculations that the underlying layer is of infinite thickness.

Formulas (8) and (11) above were derived for the case of a single-layer floor. Actually, however, most floors are multi-layer. In calculat-

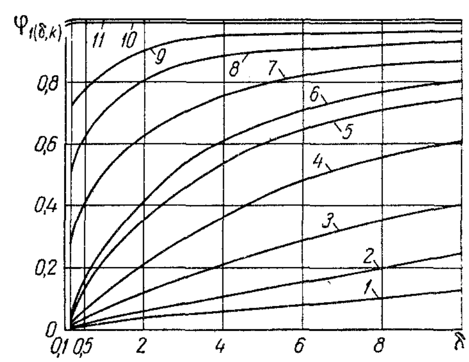

Fig. 1. $\varphi_{1(\delta,k)}$ as a function of δ: 1) k = 0. 0125; 2) 0. 025; 3) 0. 05; 4) 0. 1; 5) 0. 2; 6) 0. 25; 7) 0. 5; 8) 0. 75; 9) 1; 10) 2; 11) 3. 45.

ing the heat flux absorbed by a multi-layer floor, the multi-layer construction can be replaced by the equivalent single-layer floor.

The problem is as follows: to find the law for reducing a construction that is nonuniform in the direction of heat flow to the equivalent uniform construction, in such a way that, for identical boundary conditions, the heat flux passing through the plane x = 0 in a given time interval τ is identical. Consider the following problem.

An infinite slab of thickness l_1 made of another material is placed on a semi-infinite body. The temperature of the combined nonuniform body at the initial moment of time is zero throughout its entire volume. From time $\tau > 0$ a constant temperature $t_c > 0$ is maintained in the upper plane of the slab (x = 0). It is required to determine the heat flux through the slab surface (x = 0).

$$\frac{\partial}{\partial \tau} t_{1(x,\tau)} = a_1 \frac{\partial^2}{\partial x^2} t_{1(x,\tau)};\quad 0 \leqslant x \leqslant l_1; \tag{12}$$

$$\frac{\partial}{\partial \tau} t_{2(x,\tau)} = a_2 \frac{\partial^2}{\partial x^2} t_{2(x,\tau)};\quad l_1 \leqslant x < \infty. \tag{13}$$

The initial conditions are:

$$t_{1(x,0)} = 0;\quad 0 \leqslant x \leqslant l_1; \qquad t_{2(x,0)} = 0;\quad l_1 \leqslant x \leqslant \infty. \tag{14}$$

The boundary conditions are:

$$\frac{\partial}{\partial x} t_{1(l_1,\tau)} = \frac{\lambda_2}{\lambda_1}\frac{\partial}{\partial x} t_{2(l_1,\tau)};\qquad t_{1(0,\tau)} = t_c;\quad t_{2(\infty,\tau)} = 0. \tag{15}$$

The solutions of differential equations (12) and (13) with the initial and boundary conditions indicated have the form [2]:

$$\frac{t_{1(x,\tau)}}{t_c} = \operatorname{erfc}\left(\frac{x}{2\sqrt{a_1\tau}}\right) + h\sum_{n=1}^{\infty}(-h)^{n-1} \times \left[\operatorname{erfc}\left(\frac{2nl_1 - x}{2\sqrt{a_1\tau}}\right) - \operatorname{erfc}\left(\frac{2nl_1 + x}{2\sqrt{a_1\tau}}\right)\right]; \tag{16}$$

$$\frac{t_{2(x,\tau)}}{t_c} = \frac{2}{1+\delta}\sum_{n=1}^{\infty}(-h)^{n-1}\operatorname{erfc}\left[\frac{(x-l_1)+(2n-1)l_1\sqrt{a_2/a_1}}{2\sqrt{a_2\tau}}\right]. \tag{17}$$

Differentiating (16) and substituting the result in the heat flux formula, we obtain for x = 0:

$$q_{(0,\tau)} = \frac{t_c b_1}{\sqrt{\pi}\sqrt{\tau}}\left[1 + 2\sum_{n=1}^{\infty}(-h)^n \exp\left(-\frac{n^2 l_1^2}{a_1\tau}\right)\right]. \tag{18}$$

We shall assume that the slab and the semi-infinite body have identical thermophysical properties, i. e., $b_1 = b_2$. Then h = 0, and (18) takes the form:

$$q_{(0,\tau)} = t_c b_1/\sqrt{\pi}\sqrt{\tau}. \tag{19}$$

Formula (19) expresses the heat flux at the surface of a homogeneous semi-infinite body. It is clear from (19) that the characteristic determining the degree of thermal absorption for a homogeneous body with the above-mentioned boundary conditions is b_1. Hence the coefficient of thermal activity of the material characterizes the quantity $q_{(0,\tau)}$.

The heat flux for a two-layer floor (18) may be written in a form similar to that for the single-layer case:

$$q_{(0,\tau)} = t_c B/\sqrt{\pi}\sqrt{\tau}, \tag{20}$$

where

$$B = b_1\left[1 + 2\sum_{n=1}^{\infty}(-h)^n \exp\left(-\frac{n^2 l_1^2}{a_1\tau}\right)\right] = b_1(1 + K_{1,2}),\quad K_{1,2} = 2\sum_{n=1}^{\infty}(-h)^n \exp\left(-\frac{n^2 l_1^2}{a_1\tau}\right). \tag{21}$$

$K_{1,2}$ is a dimensionless parameter determined by the values of b_2/b_1 and $v = l_1^2/a_1\tau = 1/\mathrm{Fo}$ in Fig. 2, where the subscripts 1, 2 denote the corresponding layer.

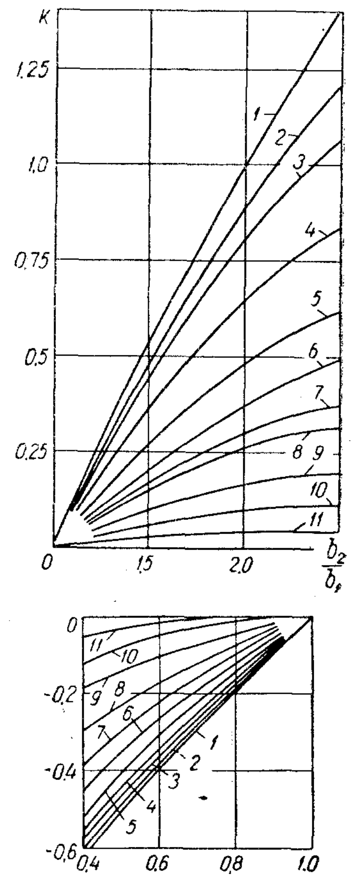

Fig. 2. $K_{1,2}$ as a function of δ: 1) $v = l_1^2/a_1\tau = 0.01$; 2) 0.05; 3) 0.1; 4) 0.2; 5) 0.4; 6) 0.6; 7) 0.8; 8) 1.0; 9) 1.5; 10) 2.0; 11) 3.0.

Formula (21) is the law for reducing a two-layer to the equivalent single-layer floor, and therefore, by analogy with (19), B is called the equivalent coefficient of thermal activity.

Equation (21) was derived for the case of an infinitely thick second layer, but the heat flux is propagated at a finite velocity; therefore the solution of (21) at a definite time will differ little from the solution for finite thickness of the second layer. The influence of the coefficient of thermal activity of the underlying layer on the value of $q_{(0,\tau)}$ depends on the parameter $l_1^2/a_1\tau$.

It may be seen from Fig. 2, that, for two-layer construction with $l_1^2/a_1\tau \geq 3$, the influence of the thermophysical properties of the material of the second layer does not exceed 5%, and therefore in practice calculations may be carried out as for a single layer. If $l_1^2/a_1\tau < 3$, the influence of the thermophysical properties of the underlying layer is taken into account by replacing the two-layer with the equivalent single-layer floor, where B is determined from (21).

This enables the following procedure to be adopted for determining the equivalent coefficient of thermal activity of multi-layer floors.

For a floor of single-layer construction:

$$B = b_1 = \sqrt{\lambda_1 c_1 \gamma_1}\ .$$

For two-layer construction, with $l_1^2/a_1\tau > 3$, $B = b_1 = \sqrt{\lambda_1 c_1 \gamma_1}$, and if $l_1^2/a_1\tau < 3$, $B = b_1(1 + K_{1,2})$.

For a floor of three-layer construction if

$$\frac{l_1^2}{a_1\tau} + \frac{l_2^2}{a_2\tau} \geqslant 3 \quad B = b_1(1 + K_{1,2}).$$

If $\frac{l_1^2}{a_1\tau} + \frac{l_2^2}{a_2\tau} < 3$, then B is determined successively: first $B_{2,3}$ is determined for the lower layers, then B is determined from (21). The parameters $K_{2,3}$ and $K_{1,2}$ are found from Fig. 2 using the relations:

$$K_{2,3} = f_1\left(\frac{b_3}{b_2}, \frac{l_2^2}{a_2\tau_2}\right) \quad \text{and} \quad K_{1,2} = f_1\left(\frac{B_{2,3}}{b_1}, \frac{l_1^2}{a_2\tau}\right), \quad \text{where} \quad \Delta\tau = \frac{l\sqrt{\tau}\,\tau}{2\sqrt{\pi a}}\ .$$

Most three-layer floors include a plastic covering 2-3 mm thick. For such construction $\Delta\tau \ll \tau$, so that the $\Delta\tau$ may be neglected and τ_2 assumed equal to τ.

A standard for the thermophysical quality of floors has been derived on the basis of the allowable heat loss from the feet, established by special health research. The amount of heat absorbed by a floor is a single-valued function of the equivalent coefficient of thermal activity, which characterizes its physical properties. The equivalent coefficient of thermal activity should therefore be adopted as a standard criterion for determining the thermophysical quality of floors.

NOTATION

t_c and t_m — temperature of foot and floor; τ — time; c — specific heat capacity; γ — volume weight of material; l — layer thickness; δ — heat accumulation ratio; q — intensity of heat flux; b — coefficient of thermal activity; τ — duration of continuous contact of foot with floor ($\tau = 0.2$ hr); $\Delta\tau$ — transmission time of heat through thickness of slab ($\Delta\tau = l\sqrt{\tau}\,\tau/2\sqrt{\pi a}$).

REFERENCES

1. Soviet Construction Norms and Specifications, Pt II, Section A, §5, 1963.
2. A. V. Lykov, Heat Conduction in Transient Processes [in Russian], Gosenergoizdat, 1948.

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Central Building Research Institute, Moscow